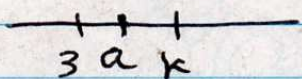


1) a) V o F?

i) si: $A = \{x \in \mathbb{R} \mid x = \frac{3n+1}{n}, n \in \mathbb{N}^*\} \rightarrow 3$ es ext(A)

I) 3 es cot(A) $\leftrightarrow \frac{3n+1}{n} \geq 3 \forall n \in \mathbb{N}^* \leftrightarrow 3n+1 \geq 3n \leftrightarrow 1 \geq 0$ (V)
 $\rightarrow 3$ es cot(A)

II) 3 es la mayor cot(A)?, veremos si: $k > 3$ puede ser cot(A)
 ¿existirá $a \in A \mid a < k$?



$\exists n \in \mathbb{N}^* \mid \frac{3n+1}{n} < k \iff k \iff 3n+1 < kn \iff$

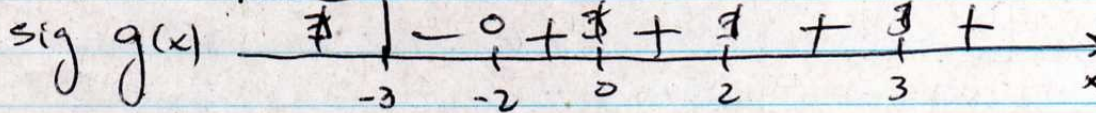
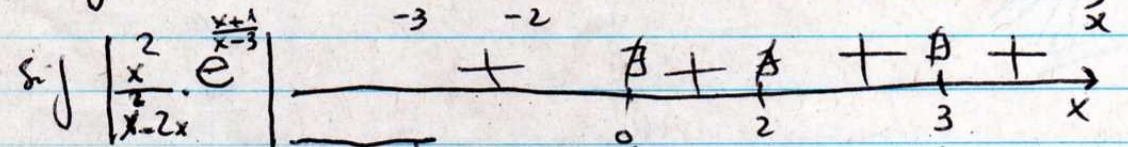
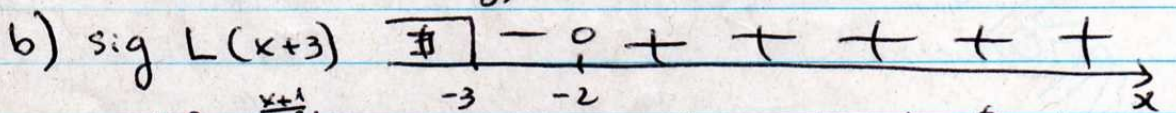
$\iff 1 < (k-3)n \iff \frac{1}{k-3} < n$ y según Arquímedes $\exists n \in \mathbb{N}^* \mid \frac{1}{k-3} \in \mathbb{R}$

$\rightarrow \exists a \in A \mid a < k$
 $\rightarrow k$ no es cot(A) $\rightarrow 3$ es la mayor cot(A)

def ext(A) \rightarrow (V)

ii) si: $x \in \mathbb{R} \mid x \neq 0, x \neq 2, x \neq 3$ $\Rightarrow \left| \frac{x^2}{x^2-2x} e^{\frac{x+1}{x-3}} \right| = \frac{x^2}{|x^2-2x|} e^{\frac{x+1}{x-3}}$ (V)

Dem $\left| \frac{x^2}{x^2-2x} e^{\frac{x+1}{x-3}} \right| = \frac{|x^2|}{|x^2-2x|} \left| e^{\frac{x+1}{x-3}} \right| \stackrel{def''}{=} \frac{x^2}{|x^2-2x|} e^{\frac{x+1}{x-3}}$



c)

$\lim_{n \rightarrow +\infty} \frac{-5e^{2n} + \sqrt{n}}{3e^{\frac{1}{n}} + \ln n}$ (Teo infinitos con $n \rightarrow +\infty$)
 Teo 1 Suma infinitos)

$\lim_{n \rightarrow +\infty} \frac{-5e^{2n} + \sqrt{n}}{3e^{\frac{1}{n}} + \ln n} = \lim_{n \rightarrow +\infty} \frac{-5e^{2n} + \sqrt{n}}{3 + \ln n}$
 Teo 2 Suma (Teo 1 Suma infinitos en división)

$= \lim_{n \rightarrow +\infty} \frac{-5e^{2n}}{\ln n} = -\infty$ (Teo infinitos con $n \rightarrow +\infty$ def órdenes)

2) a) VoF? Fundamentar

i) $a_n \sim hn^\beta$ $h+k \neq 0$ $\rightarrow a_n + b_n \sim (h+k)n^\beta$
 $b_n \sim kn^\beta$

$\lim_{n \rightarrow +\infty} \frac{a_n + b_n}{(h+k)n^\beta} = \lim_{n \rightarrow +\infty} \frac{a_n}{(h+k)n^\beta} + \frac{b_n}{(h+k)n^\beta}$

Teo Sust $\frac{h}{h+k} + \frac{k}{h+k} = \frac{h+k}{h+k} = 1$
 Teo Suma

ii) si $a_n \sim b_n \rightarrow \lim_{n \rightarrow +\infty} (a_n - b_n) = 0$ Falso
 (Contraejemplos en cuaderno)

b) i) $\lim_{n \rightarrow +\infty} \left(\frac{\sqrt{n^2 - n} + 2n}{n} \right) \sim 3n \lim_{n \rightarrow +\infty} \left| \frac{-n+1}{n} \right| =$

$= \lim_{n \rightarrow +\infty} 3n \lim_{n \rightarrow +\infty} \left(\frac{n-1}{n} - 1 \right) =$

$= \lim_{n \rightarrow +\infty} 3n \left(\frac{n-1-n}{n} \right) = \lim_{n \rightarrow +\infty} \frac{-3n}{n} = (-3)$

(3)

2) b) ii)

$$\lim_{n \rightarrow \infty} [(2n+3)e^{\frac{1}{n}} - 2n] =$$

$$= \lim_{n \rightarrow \infty} (2n e^{\frac{1}{n}} + 3 e^{\frac{1}{n}} - 2n) =$$

$$= \lim_{n \rightarrow \infty} 2n \left(e^{\frac{1}{n}} - 1 \right) + 3$$

\downarrow
 $\sim \frac{1}{n}$

$\begin{matrix} e^x \sim 1 + x \\ x \rightarrow 0 \end{matrix}$

$$= \lim_{n \rightarrow \infty} 2n \cdot \frac{1}{n} + 3 = \boxed{5}$$