1era PP 6to I 12 Mat "A" 4/10/10 Resolución (1)

1) a) V o F?

i) si $A=\{x\in\mathbb{R} / x=\frac{3n+1}{n}, n\in\mathbb{N}^*\} \rightarrow 3$ es $\underline{ext}(A)$

I) 3 es $\underline{cot}(A) \leftrightarrow \frac{3n+1}{n}\geq 3\ \forall n\in\mathbb{N}^* \leftrightarrow 3n+1\geq 3n \leftrightarrow 1\geq 0$ (V)

$\rightarrow$ 3 es $\underline{cot}(A)$

II) 3 es la mayor $\underline{cot}(A)$?, veremos si $k>3$ puede ser $\underline{cot}(A)$

¿existirá $a\in A / a<k$?

3 a k

$\updownarrow$

$\exists n\in\mathbb{N}^* / \underbrace{\frac{3n+1}{n}}_{a} < k \underset{n>0}{\leftrightarrow} 3n+1<kn \leftrightarrow$

$\leftrightarrow 1<(k-3)n \underset{k-3>0}{\longleftrightarrow} \frac{1}{k-3}<n$ y según arquímedes $\exists n\in\mathbb{N}^* > \frac{1}{k-3}\in\mathbb{R}$

$\rightarrow \exists a\in A / a<k$

$\rightarrow$ k no es $\underline{cot}(A) \rightarrow$ 3 es la mayor $\underline{cot}(A)$

$\underset{ext}{\overset{def}{\rightarrow}}$ 3 es $\underline{ext}(A) \rightarrow$ (V)

ii) si $x\in\mathbb{R}$, $x\neq 0, x\neq 2, x\neq 3 \Rightarrow \left|\frac{x^2}{x^2-2x}e^{\frac{x+1}{x-3}}\right| = \frac{x^2}{|x^2-2x|}e^{\frac{x+1}{x-3}}$ (V)

Dem

$\left|\frac{x^2}{x^2-2x}\cdot e^{\frac{x+1}{x-3}}\right| \underset{|\frac{x}{y}|=\frac{|x|}{|y|}}{\overset{|x\cdot y|=|x|\cdot|y|}{=}} \frac{|x^2|}{|x^2-2x|}\left|e^{\frac{x+1}{x-3}}\right| \overset{\text{def}}{=} \frac{x^2}{|x^2-2x|}\cdot e^{\frac{x+1}{x-3}}$

$\geq 0\ \forall x$; $>0\ \forall x\neq 3$

b) sig $L(x+3)$: $\nexists$] (−3) − 0 (−2) + + + + + x

sig $\left|\frac{x^2}{x^2-2x}\cdot e^{\frac{x+1}{x-3}}\right|$: + $\nexists$ (0) + $\nexists$ (2) + $\nexists$ (3) + x

sig $g(x)$: $\nexists$] (−3) − 0 (−2) + $\nexists$ (0) + $\nexists$ (2) + $\nexists$ (3) + x

1era PP 6to IR2 Mat "A" 4/10/10 Resolución (2)

c) $\lim\limits_{n\to+\infty} \dfrac{-5e^{2n} + \sqrt{n}}{3e^{\frac{1}{n}} + Ln}$ $\sim -5e^{2n}$ (teo infinitos con $n\to+\infty$ Teo 1 suma infinitos)

$\sqrt{n} = n^{\frac{1}{2}}$, $3e^{\frac{1}{n}} \to 3$

$=$ teo Sust x equiv en división

$\sim Ln$ (teo 2 suma)

$= \lim\limits_{n\to+\infty} \dfrac{-5e^{2n}}{Ln} = -\infty$ (teo infinitos con $n\to+\infty$ def órdenes)

2) a) V o F? Fundamentar

i) $\left.\begin{array}{l} a_n \sim h n^{\beta} \quad h+k\neq 0 \\ b_n \sim k n^{\beta} \end{array}\right\} \rightarrow a_n + b_n \sim (h+k) n^{\beta}$

$\lim \dfrac{a_n+b_n}{(h+k)n^{\beta}} = \lim \dfrac{a_n}{(h+k)n^{\beta}} + \dfrac{b_n}{(h+k)n^{\beta}}$

$a_n \sim h n^{\beta}$, $\dfrac{a_n}{(h+k)n^{\beta}} \to \dfrac{h}{h+k}$, $\dfrac{b_n}{(h+k)n^{\beta}} \sim \dfrac{k}{h+k}$

$\overset{\text{teo sust}}{\underset{\text{teo lim suma}}{=}} \dfrac{h}{h+k} + \dfrac{k}{h+k} = \dfrac{h+k}{h+k} = 1$

$h+k \neq 0$

$\rightarrow$ V

ii) si $a_n \sim b_n \rightarrow \lim(a_n - b_n) = 0$ Falso

(Contraejemplos en cuaderno)

b) i) $\lim\left(\sqrt{n^2-n} + 2n\right) L\left|\dfrac{-n+1}{n}\right| =$

$\sqrt{n^2-n} \sim n$, $\sqrt{n^2-n}+2n \sim 3n$, $\dfrac{-n+1}{n} \to -1$

$= \lim 3n\, L\left(\dfrac{n-1}{n}\right) = \lim 3n\left(\dfrac{n-1}{n} - 1\right) =$

$\dfrac{n-1}{n} \to 1$

$= \lim 3n\left(\dfrac{n-1-n}{n}\right) = \lim \dfrac{-3n}{n} = -3$

2) b) ii)

$$\lim \left[(2n+3)e^{\frac{1}{n}} - 2n\right] =$$

$$= \lim \left(2n e^{\frac{1}{n}} + \underbrace{3e^{\frac{1}{n}}}_{\downarrow\, 3} - 2n\right) =$$

$$= \lim 2n\left(\underbrace{e^{\overset{\to 0}{\frac{1}{n}}} - 1}_{\sim \frac{1}{n}}\right) + 3 \underset{u \to 0}{\overset{e^u - 1 \sim u}{=}}$$

$$= \lim 2\cancel{n} \cdot \frac{1}{\cancel{n}} + 3 = 5$$